Introduction to Matrices

Matrices are rectangular arrays of numbers arranged in the form of rows and columns. In this lesson, you will explore the various types of matrices with examples of each.

Matrix:

In order to arrange numerous numbers, mathematics provides a simple solution: matrices. A **matrix** can be defined as a rectangular grid of numbers, symbols, and expressions arranged in rows and columns. These grids are usually charted by brackets around them.

The dimensions of a matrix are represented as *R* X *C*, where *R* is the number of rows and *C* is the number of columns. This *R* X *C* notation is also called the **order** of the matrix.

Types of Matrices:

There are various types of matrices, depending on their structure. Let's explore the most common types:

Null Matrix:

A matrix that has all 0 elements is called a **null matrix**. It can be of any order. For example, we could have a null matrix of the order 2 X 3. It's also a **singular matrix**, since it does not have an inverse and its determinant is 0.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Any matrix that does have an inverse can be called a **regular matrix**.

Row Matrix:

A **row matrix** is a matrix with only one row. Its order would be 1 X *C*, where *C* is the number of columns. For example, here's a row matrix of the order 1 X 5:

$$\begin{bmatrix} 3 & 5 & 7 & 9 & 11 \end{bmatrix}$$

Column Matrix:

A **column matrix** is a matrix with only one column. It is represented by an order of *R* X 1, where R is the number of rows. Here's a column matrix of the order 3 X 1:

$$\begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$$

Square Matrix:

A matrix where the number of rows is equal to the number of columns is called a **square matrix**. Here's a square matrix of the order 2 X 2:

$$\begin{bmatrix} 4 & 9 \\ 15 & 2 \end{bmatrix}$$

Diagonal Matrix:

A **diagonal matrix** is a square matrix where all the elements are 0 except for those in the diagonal from the top left corner to the bottom right corner. Let's take a look at a diagonal matrix of order 4 X 4:

$$\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

Scalar Matrix:

A special type of diagonal matrix, where all the diagonal elements are equal is called a **scalar matrix**. We can see a 3 X 3 scalar matrix here:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Identity Matrix:

A scalar matrix whose diagonal elements are all 1 is called a **unit matrix**, or **identity matrix**.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Upper Triangular Matrix:

A square matrix where all the elements below the left-right diagonal are 0 is called an **upper triangular matrix**. Here's an upper triangular matrix of order 3 X 3:

$$\begin{bmatrix} 2 & 5 & 8 \\ 0 & 6 & 3 \\ 0 & 0 & 9 \end{bmatrix}$$

Lower Triangular Matrix:

A square matrix where all the elements above the left-right diagonal are 0 is called a **lower triangular matrix**. Here's what a lower triangular matrix of order 3 X 3 could look like:

$$\begin{bmatrix} 10 & 0 & 0 \\ 8 & 7 & 0 \\ 3 & 2 & 9 \end{bmatrix}$$

Symmetric Matrix:

A matrix whose transpose is the same as the original matrix is called a **symmetric matrix**. Only a square matrix can be a symmetric matrix. The **transpose** of a matrix is another matrix that is formed by switching the rows and columns of a given matrix. The given matrix *A* is a 3 X 3 symmetric matrix, since it's the same as its transpose A^T.

$$A = \begin{bmatrix} 5 & 1 & 9 \\ 1 & 8 & 6 \\ 9 & 6 & 5 \end{bmatrix}; A^T = \begin{bmatrix} 5 & 1 & 9 \\ 1 & 8 & 6 \\ 9 & 6 & 5 \end{bmatrix}$$